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ON THE COUPLING CONSTANTS IN β-DECAY

BY

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Synopsis.

Recent experimental data on superallowed β-transitions are used in a redetermination of the β-decay coupling constants. It is suggested that the β-decay interaction may contain an admixture of vector coupling besides the usually adopted scalar and tensor interactions.

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1. Introduction.

The improved accuracy in the experimental data on superallowed β-transitions as well as the determination of several new ft-values for superallowed $0 \rightarrow 0$ transitions permit a higher accuracy in the determination of the coupling constants in β-decay.

We shall follow the same procedure as applied earlier[1]). In the first section, we assume that no cross terms are present, which, according to recent recoil investigations[2]), means that the β-interaction is a mixture of scalar and tensor coupling only. In the second part, we consider the evidence on the possible admixture of axial vector and, especially, vector interaction.

2. Vanishing Cross Terms.

In Table I, we have collected the experimental data which we shall use. Only recent references which have not yet appeared in isotope tables are included. For the evaluation of the ft-values, the recent tables of Fermi integrals[3]) were used whenever possible; in other cases numerical integrations were performed.

Besides the mirror transitions between nuclei with closed

[1]) O. Kofoed-Hansen and A. Winther, Phys. Rev. **86**, 428 (1952).
A. Winther and O. Kofoed-Hansen, Mat. Fys. Medd. Dan. Vid. Selsk. **27**, no. 14 (1953).

[2]) J. M. Robson, Phys. Rev. **100**, 933 (1955).
Maxson, Allen, and Jentschke, Phys. Rev. **97**, 109 (1955).
W. P. Alford and D. R. Hamilton, Phys. Rev. **95**, 1351 (1954).
B. M. Rustad and S. L. Ruby, Phys. Rev. **89**, 880 (1953) and **97**, 991 (1955).
J. S. Allen and W. K. Jentschke, Phys. Rev. **89**, 902 (1953).

[3]) S. A. Moszkowski and K. M. Jantzen, UCLA Technical Report, no. 10—26—55.

TABLE 1. Data for transitions used in B, x diagrams.

Decay	$E_{\mathrm{Mev}}^{\max}$	t	ft	$\lvert\int 1\rvert^2$	$\lvert\int\vec{\sigma}\rvert^2$ Single particle	$\lvert\int\vec{\sigma}\rvert^2$ μ corrected	$(\mathrm{Weight})^{-1}$
$O^{14}\rightarrow N^{14}$	1.835 ± 8[4])	$72^{\mathrm{s}}.1 \pm 4$[4]) 99.4 pct.	3300 ± 75	2	0		75
$Al^{26}\rightarrow Mg^{26}$	3.202 ± 10[5])	$6^{\mathrm{s}}.54 \pm 10$[6])	3080 ± 80	2	0		80
$Cl^{34}\rightarrow S^{34}$	4.50 ± 3[7])	$1^{\mathrm{s}}.53 \pm 2$[8])	3110 ± 120	2	0		120
$K^{38}\rightarrow A^{38}$	5.06 ± 11[9])	$0^{\mathrm{s}}.935 \pm 25$[8])	3140 ± 400	2	0		400
$n\rightarrow p$	.782 ± 1	$12^{\mathrm{m}}.2 \pm 1.5$[10])	1220 ± 150	1	3		300
$H^{3}\rightarrow He^{3}$	.0183 ± 2	$12^{\mathrm{y}}.262 \pm 4$[11])	1060 ± 40	1	3	3.51[12]) 3.72[13]) 3.62[14])	370
$O^{15}\rightarrow N^{15}$	1.735 ± 8[15])	$123^{\mathrm{s}} \pm 2$[8])	4400 ± 100	1	1/3	0.350	100
$F^{17}\rightarrow O^{17}$	1.746 ± 6[16])	$65^{\mathrm{s}} \pm 2$[17])	2330 ± 80	1	7/5	1.373	100
$Ca^{39}\rightarrow K^{39}$	5.58 ± 8[18])	$0^{\mathrm{s}}.90 \pm 1$[8])	4650 ± 300	1	3/5	0.390	650
$Se^{41}\rightarrow Ca^{41}$	4.94 ± 5[19])	$0^{\mathrm{s}}.87 \pm 5$	2560 ± 160	1	9/7		430[20])

[4]) R. SHERR and J. B. GERHART, Phys. Rev. **91**, 909 (1953).
J. B. GERHART, Phys. Rev. **95**, 288 (1954).
SHERR, GERHART, HORIE, and HORNYAK, Phys. Rev. **100**, 945 (1955).

[5]) KINGTON, BAIR, COHN, and WILLARD, Phys. Rev. **99**, 1393 (1955).
ENDT, KLUYVER, and VAN DER LEUN, Physica **20**, 1299 (1954), and Phys. Rev. **94**, 1795 (1954).
ELBEK, MADSEN, and NATHAN, Phil. Mag. **46**, 663 (1955).
T. H. HANDLEY and W. S. LYON, Phys. Rev. **99**, 755 (1955).
KAVANAGH, MILLS, and SHERR, Phys. Rev. **97**, 248 (1955).

[6]) HASLAM, ROBERTS, and ROBB, Can. J. Phys. **32**, 361 (1954).
GREEN, HARRIS, and COOPER, Phys. Rev. **96**, 817 (1954).

[7]) W. ARBER and P. STÄHELIN, Helv. Phys. Acta **26**, 433 (1953).
P. STÄHELIN, Helv. Phys. Acta **26**, 691 (1953).
D. GREEN and J. R. RICHARDSON, Phys. Rev. **96**, 858 (1954).

[8]) R. M. KLINE and D. J. ZAFFARANO, Phys. Rev. **96**, 1620 (1954).

[9]) W. A. HUNT, Thesis, Iowa State College, 1954.
P. STÄHELIN, Helv. Phys. Acta **26**, 691 (1953).

[10]) SPIVAC, SOSNOVSKY, PROKOFIEV, and SOKOLOV, Geneva Conference. A/CONF 8/P/650 (1955).

[11]) W. M. JONES, Phys. Rev. **100**, 124 (1955).

[12]) From H^3 magnetic moment.

[13]) From He^3 magnetic moment.

[14]) Average value.

[15]) KINGTON, BAIR, COHN, and WILLARD, Phys. Rev. **99**, 1393 (1955).

[16]) C. WONG, Phys. Rev. **95**, 765 (1954).

[17]) WARREN, LAURIE, JAMES, and ERDMAN, Can. J. Phys. **32**, 563 (1954), L. KOESTER, Zeit. f. Naturf. **9a**, 104 (1954).

[18]) D. J. ZAFFARANO, priv. comm.

[19]) H. S. PLENDL and F. E. STEIGERT, Phys. Rev. **98**, 1538 (1955).

[20]) Matrix element uncertainty equated to uncertainty for Ca^{39}.

shells $\pm$ one nucleon, we have included the transitions of type $0 \rightarrow 0$, $\Delta T = 0$ (no). The Fermi matrix element, $|\int 1|^2$, for all the transitions can be determined from the assumption of charge independence of nuclear forces only[21]). Coulomb corrections are expected to be small for the light nuclei in question and will be neglected. While the Gamow-Teller matrix elements vanish for the $0 \rightarrow 0$ transitions, the matrix elements for the other transitions in Table I are expected to be given in a good approximation by the single-particle value quoted in column 6. This is supported by the fact that in most cases also the magnetic moment of these nuclei deviates only slightly from the single-particle value. A semi-empirical value for the Gamow-Teller matrix element obtained from the magnetic moment, μ, is given by[1])

$$|\int \vec{\sigma}|^2 = 4\frac{J+1}{J}\left(\frac{\mu - g_l J}{g_s - g_l}\right)^2, \tag{1}$$

where J is the nuclear spin, and g_l and g_s are the gyromagnetic ratios for orbital angular momentum and spin of the odd particle, respectively. In the following, we adopt the matrix element values of eq. (1) for the closed shell $\pm$ one nucleon transition. However, in the weight which we attribute to the transition (column 8), we include the deviation of eq. (1) from the single-particle value as an additional uncertainty besides the experimental.

We find for each β-transition a B, x line defined by

$$B = ft\left\{(1-x)|\int 1|^2 + x|\int \vec{\sigma}|^2\right\} \tag{2}$$

with

$$B = \frac{2\pi^3 \hbar^7 \ln 2}{(g_S^2 + g_T^2)\, m^5 c^4} \tag{3}$$

and

$$x = g_T^2/(g_S^2 + g_T^2), \tag{4}$$

[21]) E. Wigner and E. Feenberg, Rep. Prog. Phys. 8, 274 (1941).

where g_S and g_T are the scalar and tensor coupling constants, respectively. We use the conventional units where f is measured in units $m = c = 1$, and t in seconds.

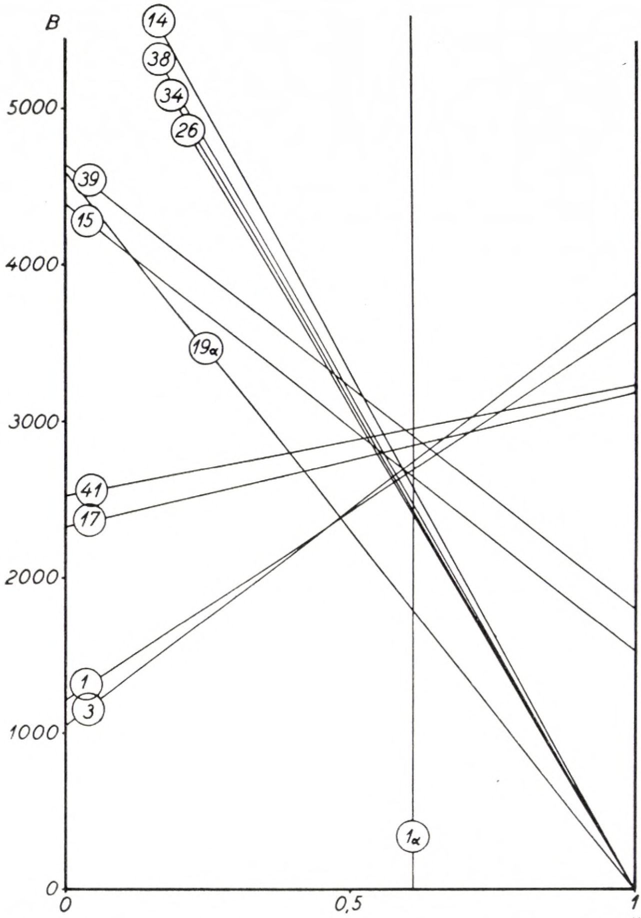

Fig. 1. The B,x diagram under the assumption of vanishing cross terms. The mass numbers of the transitions are indicated.

The B,x plot obtained from eq. (2) by means of the data in Table I is shown in Fig. 1. In this diagram, we have also included the recent correlation data from the neutron decay and Ne^{19}. For the neutron $\left|\int 1\right|^2$ and $\left|\int \vec{\sigma}\right|^2$ are known and we may therefore write for the angular correlation parameter

$$\alpha = \frac{-g_S^2 \left| \int 1 \right|^2 + \frac{1}{3} g_T^2 \left| \int \vec{\sigma} \right|^2}{g_S^2 \left| \int 1 \right|^2 + g_T^2 \left| \int \vec{\sigma} \right|^2} \tag{5}$$

$$= \frac{-(1-x)+x}{(1-x)+3\,x} \tag{6}$$

which together with the value $\alpha = 0.089 \pm 0.108$, found by Robson[2]), gives

$$x = \frac{1}{2}\frac{1+\alpha}{1-\alpha} = 0.60 \pm 0.13. \tag{7}$$

This leads to the vertical line marked 1_α in Fig. 1.

For Ne^{19} we may combine the *ft*-value with the angular correlation parameter $\alpha = -0.21 \pm 0.08$ found by Maxson *et al.*[2]) and with the $\left| \int 1 \right|^2$ value found from charge independence[21]). We may then solve eq. (5) with respect to B and x and find

$$\left.\begin{aligned} B &= ft \left| \int 1 \right|^2 (4/(1-3\,\alpha))\,(1-x) \\ &= (4600 \pm 900)\,(1-x), \end{aligned}\right\} \tag{8}$$

which is a B, x line of exactly the same type as those for the $0 \to 0$ transition, but numerically slightly inconsistent with these. This line is marked 19_α in Fig. 1.

Using the method of least squares and applying the weights given in Table I, we obtain the value

$$\left.\begin{aligned} B &= 2787 \pm 70 \\ x &= 0.560 \pm .012 \end{aligned}\right\} \tag{9}$$

for the common intersection point. The errors quoted are twice the standard error as obtained from internal consistency of the data. It should be noted that the B, x plot is not internally consistent inside the experimental errors quoted in Table I (cf. O^{14} and Al^{26}).

It is evident that systematic errors involved in the evaluation of the matrix elements may add to the errors given in eq.s (9).

The Coulomb corrections, although small, are errors of this type[22]). However, the sign is such that the inconsistency between O^{14} and Al^{26} is enlarged. Another source of systematic errors is the possible existence of cross terms.

3. Non-vanishing Cross Terms.

The limits available on the cross terms are derived from three sources: the shapes of β-spectra, the K-capture to positron ratios, and the consistency of the B, x diagram, whereas the recoil correlations are indeed very insensitive to such effects[1, 23]).

The limits obtained from β-spectrum shapes have been summarized by MAHMOUD and KONOPINSKI[24]) and by DAVIDSON and PEASLEE[25]). Also recent He^6 spectrum measurements should be taken into account[26]) as wsll as measurements of the spectra of C^{11} and F^{17} [27]). The limits in the GAMOW-TELLER interference term is quite well established in this way with the result $|g_A/g_T| < 0.05$ based especially on the He^6 spectrum. Here, g_A is the axial vector coupling constant. Information about the Fermi interference term was based solely on the N^{13} spectrum and the statements made on the vector coupling constant g_V are therefore somewhat more uncertain. KONOPINSKI and MAHMOUD conclude that $|g_V/g_s| < 0.20$. The spectra of C^{11} and F^{17} do not permit to narrow this limit (cf. Fig. 4).

The K capture to positron emission ratio for Na^{22} studied by SHERR and MILLER[28]) leads to the estimate $g_A/g_T = -0.01 \pm 0.02$.

These limits for the Fierz terms are, in Fig. 2, expressed as limits on the interference term constant b_F and b_{GT} given by

$$b_F = \frac{2\gamma g_S g_V}{g_S^2 + g_V^2}, \tag{10}$$

22) W. M. McDONALD, Princeton thesis 1955.
23) O. KOFOED-HANSEN and A. WINTHER, Phys. Rev. **89**, 526 (1953).
24) H. M. MAHMOUD and E. J. KONOPINSKI, Phys. Rev. **88**, 1266 (1952).
25) J. P. DAVIDSON and D. C. PEASLEE, Phys. Rev. **91**, 1232 (1953).
26) A. SCHWARZCHILD, priv. com.
27) C. WONG, Phys. Rev. **95**, 765 (1954).
28) R. SHERR and R. H. MILLER, Phys. Rev. **93**, 1076 (1954).

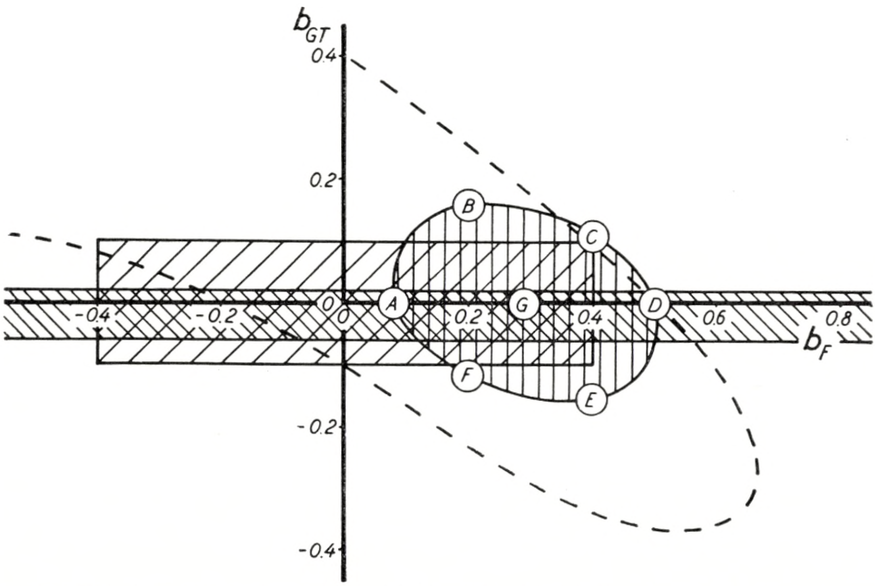

Fig. 2. The areas in the b_F, b_{GT} plane which are consistent with experimental data. B, x values in the points A to G are given in Table II.

limits from K capture to positron ratio of Na^{22}.

limits from spectral shapes.

limits from consistency of (1953) B, x plot.

limits from consistency of (1956) B, x plot.

and

$$b_{GT} = \frac{2\gamma\, g_A\, g_T}{g_A^2 + g_T^2}, \tag{11}$$

where

$$\gamma = \sqrt{1-(\alpha Z)^2}. \tag{12}$$

In this figure, we also show the older limits on possible b_F, b_{GT} values as derived from internal consistency of the B, x diagram[1]). In using the B, x diagram for such investigation we redefine

$$\left.\begin{aligned} B = ft\Big\{ &(1-x)\,(1 \pm b_F \langle 1/W \rangle_{AV}) \left| \int 1 \right|^2 \\ &+ x\,(1 \pm b_{GT} \langle 1/W \rangle_{AV}) \left| \int \vec{\sigma} \right|^2 \Big\} \end{aligned}\right\} \tag{13}$$

and

$$x = \frac{g_T^2 + g_A^2}{g_S^2 + g_V^2 + g_A^2 + g_T^2}, \tag{14}$$

where the + sign in (13) applies to β^- decay and the — sign to β^+ decay.

With the new *ft*-values of Table I, one obtains a much narrower region which is also given in Fig. 2. The limits correspond to twice the standard deviation as observed from internal consistency of the B, x diagram and coincide very closely with the points where one or more of the experimental lines show a definite inconsistency with the common B, x point in question inside the experimental errors. It is noted that inside the region the $0 \to 0$ transitions show consistent *ft*-values contrary to the case of no interference terms discussed above. Also no inconsistency with the neutron recoil correlation occurs, and the Ne^{19} correlation is in no worse agreement here than in the case of absence of Fierz terms.

TABLE II. B, x values at the b_F, b_{GT} points indicated in Fig. 2 and at $b_F = b_{GT} = 0$.

b_F, b_{GT} point	B	x
A	2750	0.553
B	2640	0.552
C	2550	0.535
D	2510	0.522
E	2630	0.518
F	2720	0.539
G	2620	0.537
0,0	2787	0.560

Of course, B and x are now functions of b_F, b_{GT} and we have given, in Table II, a sequence of values in the center and at the border of the region of consistency. It is seen that the variations of B and x are much larger than the uncertainties found for fixed values of b_F and b_{GT} (cf. eq.s (9)). In Fig. 3, we give the B, x plot corresponding to the most probable value of $(b_F, b_{GT}) = (0.29, 0)$ and, in Fig. 4, we show the Fierz plots of the spectra of C^{11} and F^{17} derived under the assumption that $b_F = 0.29$ and using the matrix element obtained from charge independence and the B, x point of Fig. 3.

If one includes the Coulomb correction as recently calcu-

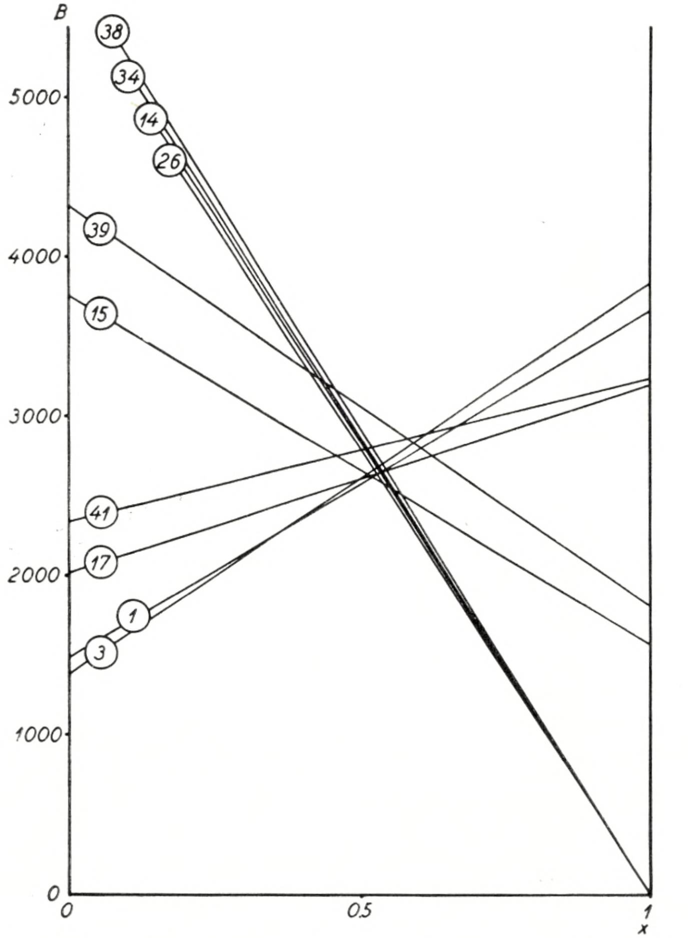

Fig. 3. The B, x diagram for the best fit obtained at $b_F = 0.29$ and $b_{GT} = 0$.

lated[22]) in the cross term investigation, this correction tends to lower B and x and to make b_F larger.

It is seen that the available material is consistent with the assumption of the presence of a small amount of vector coupling, but it should be remembered that the conclusion from the B, x plots is on the limits of the uncertainties in the experimental data as well as on the theoretical evaluation of the matrix elements.

It is interesting to note that recent experiments[29]) indicate a small difference between the spectra of Al^{25} and Al^{26}

[29]) Elbek, Madsen, and Nathan, Phil. Mag. **46**, 663 (1955).

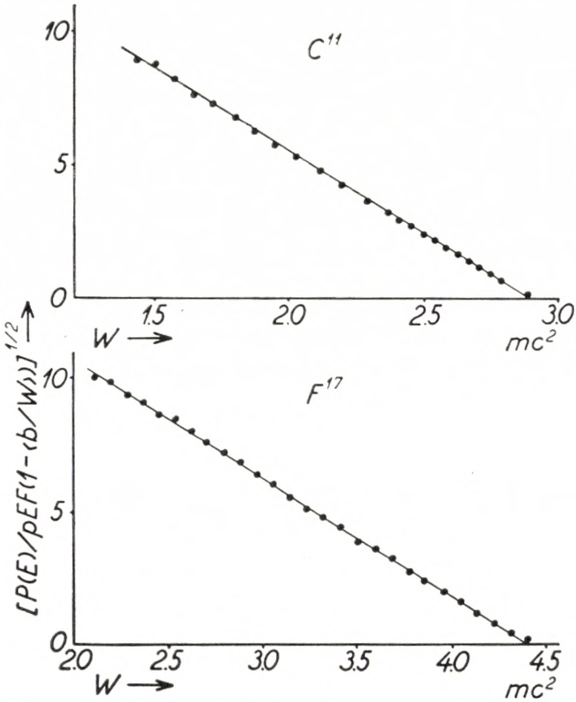

Fig. 4. Fierz plots of the C^{11} and F^{17} spectra observed by Wong[27]) using $b_F = 0.29$ and $b_{GT} = 0$.

measured under identical conditions. This could be ascribed to the above amount of cross terms even allowing for the branching in the Al^{25} decay. However, the accuracy in the spectra hardly permits definite conclusions. Thus it is to be hoped that further comparisons of β-spectra of neighbouring $0 \rightarrow 0$ and mirror transitions will be carried out. Such transitions show nearly the same maximum energy, and difference spectra might therefore be independent of scattering troubles which usually prevent accurate information about cross terms.

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